Problem Sheet 3 For Supervision in Week 10

1. Return to the last question of last week and find **all** integer solutions to the following:

   (i)★ $3m + 5n = 1$,

   (ii) $2m + 15n = 4$,

   (iii)★ $31m + 385n = 1$,

   (iv) $41m + 73n = 20$.

   (v)★ $93m + 81n = 3$,

   (vi) $697m + 527n = 13$,

   (vii)★ $533m + 403n = 52$.

2. Alison spends £11.00 on sweets for prizes in a contest. If a large box of sweets costs 90p and a small box 70p, how many boxes of each size did she buy?

3. Use Euclid's Algorithm to find all solutions to

   i) $31x \equiv 4 \bmod 41$,

   ii) $97x \equiv 2 \bmod 157$,

   iii) ★ $1679x \equiv 21 \bmod 2323$,

   iv) ★ $87x \equiv 57 \bmod 105$.

   v) ★ $31x \equiv 4 \bmod 385$,

   vi) $32x \equiv 47 \bmod 385$,

   vii) $47x \equiv 13 \bmod 73$,

   viii) $42x \equiv 90 \bmod 156$.

4. i) Find the inverse of 5 modulo 43.

   ii) Solve the following congruences

   a) $5x \equiv 17 \bmod 43$,

   b) $25x \equiv 13 \bmod 43$,

   c) $26x \equiv 41 \bmod 43$.

5. Solve the following systems of linear Diophantine equations.

i)
$$x \equiv 3 \bmod 11,$$
$$x \equiv 4 \bmod 13.$$

ii) ★
$$2x \equiv 1 \bmod 7,$$
$$4x \equiv 6 \bmod 11.$$

iii) ★
$$x \equiv 432 \bmod 527,$$
$$x \equiv 324 \bmod 697.$$

iv)
$$31x \equiv 4 \bmod 41,$$
$$47x \equiv 13 \bmod 73.$$

v)★
$$x \equiv 1 \bmod 4,$$
$$x \equiv 2 \bmod 3,$$
$$x \equiv 3 \bmod 7.$$

v)
$$5x \equiv 1 \bmod 7,$$
$$9x \equiv 4 \bmod 11,$$
$$11x \equiv 2 \bmod 13.$$

6. a) By using the method of *successive squaring*, find the remainders of the following numbers on dividing by 41.

   (i) 5^4, (ii) 5^{16}, (iii) 5^{64}.

   b) Use the answers to part (a) to find an $n \in \mathbb{N}$ such that $5^n \equiv 1 \bmod 41$.

   c) Use part (b) to solve $25x \equiv 7 \bmod 41$.

7. What are the remainders when 3^{40} and 40^{35} are divided by 11?

   Prove that $3^{40} + 40^{35}$ is divisible by 11.